РЕШЕНИЕ ТРИГОНОМЕТРИЧЕСКИХ УРАВНЕНИЙ.

1) $2\cos x + \sqrt{3} = 0$

$2 \cdot \cos x = -\sqrt{3}$

$\cos x = -\frac{\sqrt{3}}{2}$

$x = \pm \arccos\left(-\frac{\sqrt{3}}{2}\right) + 2\pi n$

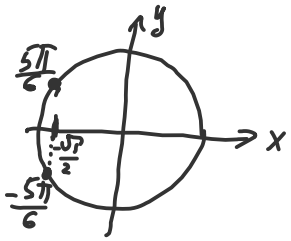
$x = \pm\left(\pi - \arccos\frac{\sqrt{3}}{2}\right) + 2\pi n$

$x = \pm\left(\pi - \frac{\pi}{6}\right) + 2\pi n$

$x = \pm\frac{5\pi}{6} + 2\pi n, n \in Z$

$\cos x = a$

$x = \pm \arccos a + 2\pi n, n \in Z$

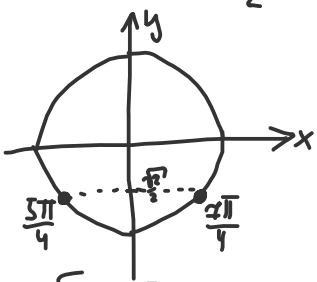


2) $\sqrt{2} \cdot \sin x + 1 = 0$

$\sqrt{2} \cdot \sin x = -1$

$\sin x = -\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$\sin x = -\frac{\sqrt{2}}{2}$



$\left[\begin{array}{l} x = \frac{5\pi}{4} + 2\pi n, n \in Z \\ x = \frac{7\pi}{4} + 2\pi n, n \in Z \end{array}\right.$

$\sin x = a$

$x = (-1)^n \cdot \arcsin a + \pi n, n \in Z$

$x = (-1)^n \cdot \arcsin\left(-\frac{\sqrt{2}}{2}\right) + \pi n$

$x = (-1)^n \cdot \left(-\arcsin\frac{\sqrt{2}}{2}\right) + \pi n$

$x = (-1)^n \cdot \left(-\frac{\pi}{4}\right) + \pi n$

$x = (-1)^{n+1} \cdot \frac{\pi}{4} + \pi n, n \in Z$

3) $\operatorname{tg} x + \sqrt{3} = 0$

$\operatorname{tg} x = -\sqrt{3}$

$x = \operatorname{arctg}(-\sqrt{3}) + \pi n, n \in Z$

$x = -\operatorname{arctg}\sqrt{3} + \pi n$

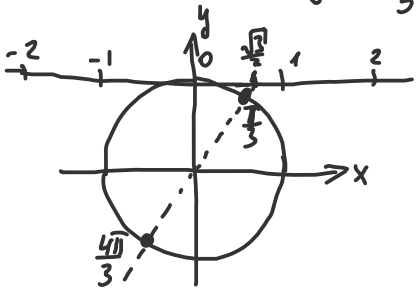
$x = -\frac{\pi}{3} + \pi n, n \in Z$

4) $\sqrt{3} \cdot \operatorname{ctg} x - 1 = 0$

$\sqrt{3} \cdot \operatorname{ctg} x = 1$

$\operatorname{ctg} x = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$\operatorname{ctg} x = \frac{\sqrt{3}}{3}$



$x = \frac{\pi}{3} + \pi n, n \in Z$

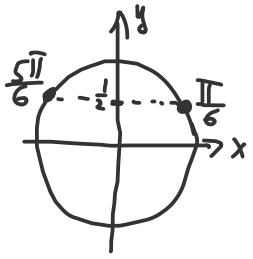
5) $\sin\frac{x}{4}=\frac{1}{2}$

$\frac{x}{4}=(-1)^n\cdot\arcsin\frac{1}{2}+\pi n,\ n\in Z$

$\frac{x}{4}=(-1)^n\cdot\frac{\pi}{6}+\pi n,\ n\in Z \quad |\cdot 4$

$x=(-1)^n\cdot\frac{\pi}{6}\cdot 4+\pi n\cdot 4$

$x=(-1)^n\cdot\frac{2\pi}{3}+4\pi n,\ n\in Z$



$\frac{x}{4}=2$

$x=2\cdot 4$

$x=8$

$(3\cdot 2+4)\cdot 4=40$

$3\cdot 2\cdot 4+4\cdot 4=40$

24 16

$3\cdot 4\cdot 2\cdot 4+4\cdot 4$

12 8 16

$\begin{cases}\frac{x}{4}=\frac{\pi}{6}+2\pi n\\ \frac{x}{4}=\frac{5\pi}{6}+2\pi n\end{cases}$ $\begin{cases}x=\frac{2\pi}{3}+8\pi n,\ n\in Z\\ x=\frac{10\pi}{3}+8\pi n,\ n\in Z\end{cases}$

6) $\cos 4x=0$ $\quad \cos x=0 \quad x=\frac{\pi}{2}+\pi n,\ n\in Z$

$4x=\frac{\pi}{2}+\pi n \quad |:4$

$x=\frac{\pi}{8}+\frac{\pi}{4}n,\ n\in Z$

7) $\operatorname{ctg}\left(-\frac{x}{2}\right)=1$ $\quad \operatorname{ctg}x=a$

$x=\operatorname{arcctg}a+\pi n,\ n\in Z$

$-\frac{x}{2}=\operatorname{arcctg}1+\pi n$

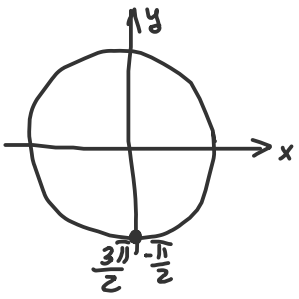
$-\frac{x}{2}=\frac{\pi}{4}+\pi n \quad |\cdot(-2)$

$x=\frac{\pi}{4}\cdot(-2)+\pi n\cdot(-2)$

$x=-\frac{\pi}{2}-2\pi n,\ n\in Z$

$x=-\frac{\pi}{2}+2\pi n,\ n\in Z$

$x=\frac{3\pi}{2}+2\pi n,\ n\in Z$



8) $2\cdot\cos\left(\frac{x}{2}-\frac{\pi}{6}\right)=\sqrt{3} \quad |:2$

$\cos\left(\frac{x}{2}-\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}$ $\quad \cos x=a$

$x=\pm\arccos a+2\pi n,\ n\in Z$

$\frac{x}{2}-\frac{\pi}{6}=\pm\arccos\frac{\sqrt{3}}{2}+2\pi n,\ n\in Z$

$\frac{x}{2}-\frac{\pi}{6}=\pm\frac{\pi}{6}+2\pi n,\ n\in Z$

$\begin{cases}\frac{x}{2}-\frac{\pi}{6}=\frac{\pi}{6}+2\pi n\\ \frac{x}{2}-\frac{\pi}{6}=-\frac{\pi}{6}+2\pi n\end{cases}$ $\begin{cases}\frac{x}{2}=\frac{\pi}{6}+\frac{\pi}{6}+2\pi n\\ \frac{x}{2}=-\cancel{\frac{\pi}{6}}+\cancel{\frac{\pi}{6}}+2\pi n\end{cases}$ $\begin{cases}\frac{x}{2}=\frac{\pi}{3}+2\pi n \quad |\cdot 2\\ \frac{x}{2}=2\pi n \quad |\cdot 2\end{cases}$

$\begin{cases}x=\frac{2\pi}{3}+4\pi n,\ n\in Z\\ x=4\pi n,\ n\in Z\end{cases}$

9) $2 \cdot \sin\left(3x - \frac{\pi}{4}\right) = -\sqrt{2}$

$\sin\left(3x - \frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

$3x - \frac{\pi}{4} = (-1)^n \cdot \arcsin\left(-\frac{\sqrt{2}}{2}\right) + \pi n, n \in \mathbb{Z}$

$3x - \frac{\pi}{4} = (-1)^n \cdot \left(-\frac{\pi}{4}\right) + \pi n, n \in \mathbb{Z}$

$3x - \frac{\pi}{4} = (-1)^{n+1} \cdot \frac{\pi}{4} + \pi n, n \in \mathbb{Z}$

$3x = (-1)^{n+1} \cdot \frac{\pi}{4} + \frac{\pi}{4} + \pi n \quad | :3$

$x = (-1)^{n+1} \cdot \frac{\pi}{12} + \frac{\pi}{12} + \frac{\pi}{3} n, n \in \mathbb{Z}$

10) $\sqrt{3} \cdot tg\left(\frac{x}{3} + \frac{\pi}{3}\right) = 3$

$tg\left(\frac{x}{3} + \frac{\pi}{3}\right) = \frac{3}{\sqrt{3}}$ $\qquad \frac{3}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\cancel{3}\sqrt{3}}{\cancel{3}} = \sqrt{3}$

$tg\left(\frac{x}{3} + \frac{\pi}{3}\right) = \sqrt{3}$ $\qquad tg\otimes = a \quad \otimes = arctg\, a + \pi n, n \in \mathbb{Z}$

$\frac{x}{3} + \frac{\pi}{3} = arctg\sqrt{3} + \pi n, n \in \mathbb{Z}$

$\frac{x}{3} + \frac{\pi}{3} = \frac{\pi}{3} + \pi n$

$\frac{x}{3} = \pi n \quad | \cdot 3$

$x = 3\pi n, n \in \mathbb{Z}$

11) $\sin 2x \cdot \cos 2x = -\frac{1}{4} \quad | \cdot 2$ $\qquad \sin 2\alpha = 2 \cdot \sin\alpha \cdot \cos\alpha$

$2 \cdot \sin 2x \cdot \cos 2x = -\frac{1}{2}$ $\qquad 2 \cdot 2x = 4x$

$\sin 4x = -\frac{1}{2}$

$4x = (-1)^n \cdot \arcsin\left(-\frac{1}{2}\right) + \pi n, n \in \mathbb{Z}$

$4x = (-1)^n \cdot \left(-\frac{\pi}{6}\right) + \pi n$

$4x = (-1)^{n+1} \cdot \frac{\pi}{6} + \pi n \quad | :4$

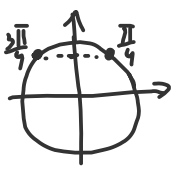
$x = (-1)^{n+1} \cdot \frac{\pi}{24} + \frac{\pi}{4} n, n \in \mathbb{Z}$

12) $\cos\left(\frac{\pi}{2}-x\right)-\sin(\pi+x)=\sqrt{2}$

$\sin x+\sin x=\sqrt{2}$

$2\sin x=\sqrt{2}$

$\sin x=\frac{\sqrt{2}}{2}$



$x=(-1)^n\cdot\frac{\pi}{4}+\pi n, n\in Z$

$\left[\begin{array}{l} x=\frac{\pi}{4}+2\pi n, n\in Z \\ x=\frac{3\pi}{4}+2\pi n, n\in Z \end{array}\right.$

13) $\sin(\pi+x)+\cos\left(\frac{\pi}{2}+x\right)=\sqrt{3}$

$-\sin x-\sin x=\sqrt{3}$

$-2\sin x=\sqrt{3} \quad |:(-2)$

$\sin x=-\frac{\sqrt{3}}{2}$

$x=(-1)^n\cdot\arcsin\left(-\frac{\sqrt{3}}{2}\right)+\pi n, n\in Z$

$x=(-1)^n\cdot\left(-\frac{\pi}{3}\right)+\pi n$

$x=(-1)^{n+1}\cdot\frac{\pi}{3}+\pi n, n\in Z$

14) $6\cos^2 x+\cos x-1=0$

$6\cdot(\cos x)^2+\cos x-1=0$

$t=\cos x$

$6t^2+t-1=0$

$D=1-4\cdot6\cdot(-1)=1+24=25$

$t_{1,2}=\frac{-1\pm5}{12}=\frac{1}{3};-\frac{1}{2}$

$\cos x=\frac{1}{3}$ $\quad$ $\cos x=-\frac{1}{2}$

$x=\pm\arccos\frac{1}{3}+2\pi n, n\in Z$

$x=\pm\frac{2\pi}{3}+2\pi n, n\in Z$

15) $\cos\frac{\pi x}{3}=\frac{\sqrt{3}}{2}$

Решите уравнение. В ответ запишите наименьший положительный корень.

$\frac{\pi x}{3}=\pm\arccos\frac{\sqrt{3}}{2}+2\pi n$

$\frac{\pi x}{3}=\pm\frac{\pi}{6}+2\pi n \quad |\cdot3$

$\pi x=\pm\frac{\pi}{2}+6\pi n, n\in Z \quad |:\pi$

$x=\pm\frac{\pi^1}{2\cdot\pi}+\frac{6\pi n}{\pi}$

$x=\pm\frac{1}{2}+6n, n\in Z$

Ответ: 0,5

$n=0$ $\quad x=\pm\frac{1}{2}$

$n=1$ $\quad x=\pm\frac{1}{2}+6$ $\quad x=6,5$ $\quad x=5,5$

$n=-1$ $\quad x=\pm\frac{1}{2}-6$ $\quad x=-5,5$ $\quad x=-6,5$